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Degrees of Freedom and Characteristic Modes

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Summary

Multiple beams are frequently used in modern communication systems. The number of such beams is related to the spatial degrees of freedom. In this paper, a scattering-based formulation of characteristic mode analysis is used to determine the degrees of freedom of arbitrary-shaped objects. It is shown that the average shadow area of electrically large objects can be used to estimate the number of dominating characteristic modes for that object.

1 Introduction

Electromagnetic degrees of freedom (DoF) is fundamental in design and optimization of antennas, wireless communication systems, and electromagnetic scattering problems [1, 2, 3, 4]. DoF can be used to determine the number of parameters that can be adjusted to achieve a desired performance, *e.g.*, many DoF allows for improved control over the radiation pattern of antennas and is related to the maximum directivity [4]. In communication systems, it is linked to the number of channels or spatial dimensions that can be used to transmit signals, achieve high data rates, and improved reliability in *e.g.*, multiple-input multiple-output (MIMO) systems and intelligent surfaces. In scattering, the number of DoF is associated with control of scattering and radar cross-sections. The number of electromagnetic DoF generally increases with the size of the object. For example, small antennas have only a few DoF, whereas large antenna arrays have many DoF. In this presentation, scattering theory is combined with characteristic mode analysis to determine the DoF of arbitrary-shaped objects. It is shown that the average shadow area of an electrically large object can be used to estimate the number of dominating characteristic modes for that object.

Characteristic mode (CM) analysis is a popular tool for antenna analysis available in many simulation packages [5]. Here, a scattering-based formulation of CM analysis [6] is used to determine the DoF of arbitrary-shaped objects. We define dominant CMs as modes with characteristic values $|\lambda_n| \leq 1$. For electrically large objects, the number of dominant CMs is further estimated from the average shadow area of the object which follows from the extinction paradox (or shadow scattering) [7]. This leads to a simple estimate of the DoF generalizing classical estimates [1, 2, 4] based on spherical wave expansions and waveguide theory to arbitrary shapes.

References

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